PERGAMON

International Journal of Solids and Structures 37 (2000) 4033–4035

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Letter to the Editor

Comments on "Accurate Equations of Laminated Composite Deep Thick Shells", Int. J. Solids Structures Vol. 36, No. 19, pp. 2917–2941 (1999) by M.S. Qatu

This author noticed with interest the recent paper by Qatu (1999) that presents a new uniform shear deformable general shell theory (FSDT). Like in Leissa and Chang (1996), the theory includes consistently the appropriate $1+z/R$ terms into the definition of its force and moment resultants. As it compares with Leissa and Chang (1996) FSDT, however, Qatu (1999) FSDT further contains some additional, higher-order, terms in its transverse shear strain components whereas it claims most accurate representation of the force and moment resultants by carrying out the denoted integrations exactly.

In order to test the reliability of his two-dimensional shell theory, Qatu (1999) considered and solved the free vibration problem of a certain class of simply supported cylindrical and spherical shell panels. Then, by choosing the value 5/6 for the shear correction factors involved in his FSDT, he compared its numerical results against corresponding three-dimensional elasticity results due to Bhimaraddi (1991). Moreover, he also compared these results with corresponding results based on certain uniform (FSDT) and parabolic (HSDT) shear deformable shell theories that make and do not make use of shear correction factors, respectively (Bhimaraddi, 1984, 1987, 1991; Librescu et al., 1989).

It should be noted in this connection that Librescu et al. (1989) FSDT and HSDT are shear deformable analogues of the classical Love-type shell theory, in the sense that they both neglect the aforementioned $1+z/R$ terms in the definition of their force and moment resultants. On the other hand, Bhimaraddi included the $1+z/R$ terms in the definition of the force and moment resultants of both his FSDT (Bhimaraddi, 1991) and HSDT (Bhimaraddi, 1984, 1987). He, however, did not discuss at all the degrees of the relevant approximations made during the derivation of the corresponding shell-type constitutive equations, whereas he employed the value $\pi^2/12$ for the shear correction factors involved in his FSDT. Hence, this author tends to believe that, as far as comparisons of two-dimensional shell theories are concerned, the only clear "competitor" of Qatu (1999) FSDT is the HSDT due to Bhimaraddi (1984, 1987), which also included the $1+z/R$ terms in the definition of its force and moment resultants. Due, however, to the form of its shell-type constitutive equations, this author further believes that the corresponding integrations denoted in the force and moment resultants of Bhimaraddi (1984, 1987) HSDT were not carried out exactly, in the sense claimed in Qatu (1999) FSDT.

As was mentioned on the other hand in a relevant brief communication (Soldatos, 1997), Bhimaraddi (1991) three-dimensional elasticity analysis is essentially an extension of a corresponding successive approximation dynamic analysis that was published slightly earlier by Soldatos and Hadjigeorgiou (1990). Moreover, in a series of relevant publications (Soldatos and Hadjigeorgiou, 1990; Soldatos 1991; Ye and Soldatos, 1994a,b; Soldatos and Ye, 1995), this author and his co-workers confirmed that this successive approximation approach is essentially exact, in the sense that it practically produces the results obtained by means of other analytical methods. There were however two cases in which Ye and Soldatos (1994a,b) failed to reproduce the results obtained elsewhere, these being related to Bhimaraddi

PII: S0020-7683(99)00233-4

(1991) dynamic analysis and its corresponding static equivalent (Bhimaraddi and Chandrashekhara, 1992). In Soldatos (1997) was further shown that, as far as the free vibrations of cylindrical shells are concerned, Wu et al. (1996) and more recent relevant elasticity results, which were based on an "exact" asymptotic analysis of three-dimensional elasticity equations, were practically identical to those obtained by Ye and Soldatos (1994a). Hence, Bhimaraddi's (1991) results were essentially proved to be inaccurate, at least as far as the free vibrations of cylindrical shells are concerned.

Under these considerations, Table 1 employs the cylindrical shell panels used in Table 5 of Qatu (1999) and compares their fundamental frequency parameters obtained on the basis of (i) all of the aforementioned three-dimensional elasticity investigations (Bhimaraddi, 1991; Ye and Soldatos, 1994a; Wu et al., 1996); (ii) Qatu (1999) FSDT and (iii) Bhimaraddi (1984, 1987) HSDT. The geometrical notation employed in Table 1 is that used in (Ye and Soldatos, 1994a; Soldatos, 1997) whereas the material properties of the shell panels employed can be found in all (Bhimaraddi, 1991; Ye and Soldatos, 1994a; Wu et al., 1996; Qatu, 1999). Keeping the pattern adopted in (Ye and Soldatos, 1994a), enclosed in parentheses are the approximate absolute errors observed in Bhimaraddi's (1991) elasticity results. Although in some cases of shallow panels this error is relatively small, it is continually increasing with increasing the R/L_x ratio and becomes as high as 2.7% for relatively deep panels ($R/L_x=1$).

The comparisons performed in Table 1 show that all the frequencies obtained on the basis of either Qatu FSDT or Bhimaraddi HSDT agree very well with the elasticity results that are due to Ye and Soldatos (1994a) and Wu et al. (1996). Moreover, in all cases, the FSDT predicts lower whereas the HSDT predicts higher fundamental frequencies than the correct elasticity solutions do. Despite that Qatu FSDT predictions appear always to be closer to the correct elasticity results than the HSDT do, the corresponding discrepancies are not as high as appeared to be in (Qatu, 1999) and, in all cases, are certainly smaller than the error observed in Bhimaraddi (1991) three-dimensional elasticity predictions.

The above discussion raises finally a question on whether a most accurate representation of the force and moment resultants of Bhimaraddi (1984, 1987) HSDT, in the sense claimed by Qatu (1999), can improve the accuracy of its results beyond the accuracy level of Qatu (1999) FSDT. This author tends

Table 1
Comparison of fundamental frequency parameters $\Omega=\omega L^2_x(\rho/E_2h^2)^{1/2}$ of two-layered regular antisymmetric cross-ply laminated cylindrical panels ($h/L_x=0.1$)

R/L_x	Three-dimensional	Analysis	FSDT (Qatu, 1999)	HSDT
1	Wu et al. (1996)	10.6875		
	Ye and Soldatos (1994)	10.6973	10.667	10.9189
	Bhimaraddi (1991)	10.4085 (2.7%)		
2	Ye and Soldatos (1994)	9.4951	9.4428	9.5664
	Bhimaraddi (1991)	9.3627 (1.4%)		
4	Ye and Soldatos (1994)	9.1155	9.0731	9.1506
	Bhimaraddi (1991)	9.0613 (0.6%)		
5	Wu et al. (1996)	9.0573		
	Ye and Soldatos (1994)	9.0616	9.0221	9.0953
	Bhimaraddi (1991)	9.0200 (0.5%)		
10	Wu et al. (1996)	8.9740		
	Ye and Soldatos (1994)	8.9778	8.9446	9.0150
	Bhimaraddi (1991)	8.9564 (0.2%)		
20	Ye and Soldatos (1994)	8.9477	8.9194	
	Bhimaraddi (1991)	8.9341 (0.2%)		8.9904
∞	Ye and Soldatos (1994)	8.9248	8.9001	8.9761
	Bhimaraddi (1991)	8.9179 (0.08%)		

to believe that this could well be the case, particularly when consideration is taken of the fact that the performance of any FSDT depends on the values of the transverse shear correction factors involved.

It could be noted on the other hand that simple, plate-type choices of these correction factors, like $\pi^2/12$, 5/6 or 2/3, have always been found quite adequate for the vibration study of homogeneous and laminated composite circular cylindrical shells, on the basis of a FSDT. Although this author does not know whether this argument is still valid when dealing with the vibrations of spherical shells or other types of shells of revolution, it was found that it was not valid when vibrations of cylindrical shells having a non-circular cross-section are investigated (Soldatos, 1986). In such a case [see also (Soldatos, 1999)], the values of the shear correction factors should further account for the type of the non-circular cross-section considered as well as for its divergence from the cross-section of a corresponding circular cylindrical shell. This however appears to be a rather impossible task for the shear correction factors which, regardless of the FSDT considered, cannot transfer the appropriate information to the relevant two-dimensional differential equations of motion even in the case of a homogeneous isotropic oval cylindrical shell (Soldatos, 1986, 1999). Since, on the other hand, a corresponding Love-type HSDT (Soldatos, 1986) was found to successfully avoid this kind of difficulty, this author suggests that higher order shear deformable theories than FSDT ones should be preferred when dealing with the analysis of shells with complicated geometrical configurations.

References

Bhimaraddi, A., 1984. A higher-order theory for free vibration analysis of circular cylindrical shells. Int. J. Solids Struct. 20, 623–630.

Bhimaraddi, A., 1987. Static and transient response of cylindrical shells. Thin-Wall. Struct. 5, 157–179.

Bhimaraddi, A., 1991. Free vibration analysis of doubly curved shallow shells on rectangular planform using three-dimensional elasticity theory. Int. J. Solids. Struct. 27, 897–913.

Bhimaraddi, A., Chandrashekhara, K., 1992. Three-dimensional elasticity solution for static response of simply supported orthotropic cylindrical shells. Compos. Struct. 20, 227–235.

Leissa, A.W., Chang, J.D., 1996. Elastic deformation of thick, laminated composite shells. Comp. Struct. 35, 153–170.

Librescu, L., Khdeir, A.A., Frederick, D., 1989. A shear-deformable theory for laminated composite shallow shell-type panels and their response analysis I: free vibration and buckling. Acta Mech. 76, 1–33.

Qatu, M.S., 1999. Accurate equations for laminated composite deep thick shells. Int. J. Solids Struct. 36, 2917–2941.

Soldatos, K.P., 1986. On thickness shear deformation theories for the dynamic analysis of non-circular cylindrical shells. Int. J. Solids Struct. 22, 625–641.

Soldatos, K.P., 1991. An iterative solution of a Bessel's equation based on torsional vibrations of orthotropic hollow cylinders. J. Sound Vibr. 151, 149–152.

Soldatos, K.P., 1997. Some comments on "An asymptotic theory for dynamic response of doubly curved laminated shells". Int. J. Solids Struct. 34, 3903–3905.

Soldatos, K.P., 1999 The mechanics of cylindrical shells with non-circular cross-section: a survey, Appl. Mech. Rev. 52, 237–274.

Soldatos, K.P., Hadjigeorgiou, V.P., 1990. Three-dimensional solution of the free vibration problem of homogeneous isotropic cylindrical shells and panels. J. Sound Vibr. 137, 369–384.

Soldatos, K.P., Ye, J.Q., 1995. Axisymmetric static and dynamic analysis of laminated hollow cylinders composed of monoclinic elastic layers. J. Sound Vibr. 184, 245–259.

Wu, C.-P., Tarn, J.-Q., Chi, S.-M., 1996. An asymptotic theory for dynamic response of doubly curved laminated shells. Int. J. Solids Struct. 33, 3813–3841.

Ye, J.Q., Soldatos, K.P., 1994a. Three-dimensional vibrations of laminated cylinders and cylindrical panels with a symmetric or an antisymmetric cross-ply lay-up. Compos. Engng 4, 429–444.

Ye, J.Q., Soldatos, K.P., 1994b. Three-dimensional stress analysis of orthotropic and cross-ply laminated hollow cylinders and cylindrical panels. Comput. Meth. Appl. Mech. Engng 117, 331–351.

Kostas P. Soldatos*

University of Nottingham, Pope Building, Nottingham NG7 2RD, UK

E-mail address: etzkps@thmech.nottingham.ac.uk

* Fax: + 44-0155-951-3837.